

Unlocking Mathematical Minds: Foundational Numeracy in Piagetian Perspective

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Abstract

The National Educational Policy 2020 has underlined the importance of early education and underscores the recognition that a robust foundation in reading, writing and arithmetic is indispensable for subsequent academic success and overall cognitive development. It is proven that by the age of 5, the synaptic connections are at its peak and 85% of human brain development takes place. As numeracy and mathematics learning are linked to one's cognitive aspect significantly, 'Foundational Numeracy' is a critical aspect of early childhood education. This article is an attempt to explore the teaching and learning at the foundational stage in light of the cognitive development theory of Piaget, in order to discuss their relevance to mathematical understanding and their application for curriculum designing and implementing effective pedagogical strategies.

Key words: Foundational Numeracy, FLN, Pedagogy of Mathematics, Piaget's Theory of Cognitive Development.

Introduction

It is psychologically proven that "Over 85% of a child's cumulative brain development occurs prior to the age of 6." Therefore, it is important to start nourishing and stimulating the brain through appropriate care and education at an early age. Recognising this need, NEP 2020 has included education of children aged between 3-6 years in the current formal education system as "Early Childhood Care and Education (ECCE)".

The objective of introducing ECCE was to strengthen learning and health care in early years of child development. The main focus areas of ECCE, in NEP2020, are Physical and Motor-Development, Cognitive Development, Socio-Emotional-Ethical-Development, Cultural/Artistic-Development, Development of Communication & Language, and Numeracy.

Some other objectives of ECCE are to develop social capabilities, sensitivity and ethical and moral values in children from an early age.

NEP stressed on the urgent need of attaining "Foundational Literacy and Numeracy (FLN)", i.e., "The ability to read and write, and perform basic operations with numbers". As the current condition of FLN in the country is quite miserable, NEP called it a "learning crisis", as a huge proportion of elementary students are lacking in basic language and numeracy (National Education Policy 2020).

"National Achievement Survey 2021" shows that in most of the states/UTs, the average achievement score of Grade 3 students in Mathematics is significantly below that of the overall achievement score and is subsequently decreased for higher standard students, i.e., grades 5,8 and 10 students. Also, students' average achievement scores

in Mathematics at all the levels were the lowest among all the disciplines examined in that achievement survey test.

Therefore, the results of NAS (2021) also express the urgent need to find the existing learning gaps and work for learning enhancement for foundational numeracy at all levels.

Research Questions

1. How do cognitive characteristics of each stage of Piaget's theory of cognitive development affect the learner's ability to grasp mathematical principles?
2. What teaching strategies can be aligned with Piaget's developmental stages to enhance mathematics learning outcomes?

Research Objectives

1. To analyse how different cognitive features of different cognitive stages of Piaget's theory of cognitive development (such as conservation, reversibility, and abstract reasoning) influence mathematics learning.
2. To propose stage-appropriate teaching-learning strategies for effective mathematics learning.

Literature Review

Recent policy-driven studies emphasise the urgency of foundational numeracy. Swargiary (2024) examined India's NEP 2020 and NIPUN Bharat initiative, identifying challenges such as regional disparities and pandemic-related learning losses. The NAS (NCERT, 2021) highlights that Grade 3 students in India possess basic numeracy skills but face challenges in conceptual understanding and problem-solving; the survey underscores the need for developmentally appropriate pedagogy,

stronger teacher support, and targeted interventions to ensure all children build a robust foundation in mathematics. The ASER Report (Pratham NGO, 2024) documented recovery in basic arithmetic post-COVID but it also shows that 66.3% of Class 3 students and 70% of Class 5 students are still unable to perform simple arithmetic calculations, which describes the condition of numeracy at foundational level. UNESCO (2021) and the World Bank (2025) also highlight global concerns, framing foundational numeracy as a prerequisite for reducing "learning poverty".

Jean Piaget's theory of cognitive development provides a foundational framework for understanding how children learn and acquire mathematical concepts. Piaget's stage-based model emphasizes that learning occurs through active construction of knowledge. Pakpahan and Saragih (2022) reaffirm that instruction must align with cognitive readiness, cautioning against introducing abstract concepts prematurely. Walter (1960) demonstrated empirically that logical thinking develops progressively, with adolescence marking the transition to abstract reasoning essential for algebra and advanced problem-solving. Ojose (2008) applied Piaget's theory to mathematics instruction, highlighting conservation and reversibility as critical for arithmetic learning. More recent works like Ondog and Kilag (2024) and Rabillas et al. (2024) extend Piagetian principles to contemporary classrooms, proposing constructivist frameworks for early numeracy and stage-appropriate scaffolding.

Foundational numeracy skills such as counting, number recognition, and simple operations, are strongly linked to later mathematical achievement. Gelmen, Meck, and Merkin (1986) showed that even young children demonstrate rudimentary numerical competence, challenging Piaget's timeline by

revealing proto-mathematical skills in the pre-operational stage. Fuson (1988) further analysed children's counting strategies, noting progression from rote counting to cardinality and flexible problem-solving.

Constructivist pedagogy, rooted in Piaget's theory, emphasises active exploration and inquiry. Martin (1997) advocated for experimental learning, while Clements and Sarama (2021) developed learning trajectories to guide early math instructions. Seigler and Braithwaite (2016) demonstrated that numerical development benefits from structured, developmentally appropriate interventions. Boaler (2016) argued for fostering mathematical mindset through creative problem-solving, while Van de Walle et al. (2021) emphasised developmental teaching strategies. Anthony and Walshaw (2009) and Mulligan and Mitchelmore (2009) reinforced the importance of effective pedagogy in mathematics education, stressing hands-on activities, scaffolding, and contextualised learning.

Policy frameworks such as India's NEP 2020 (GOI, 2020) and global initiatives by OECD and UNICEF emphasise foundational numeracy as a global priority. Together, these works highlight the enduring relevance of Piaget's theory in teaching-learning of mathematics at the foundational stage.

Methodology

This study adopts a qualitative conceptual research design, grounded in theoretical analysis and interpretive synthesis of peer-reviewed literature. This paper does not involve empirical data collection; rather it aims to critically examine and re-interpret Piaget's Theory and its significance in teaching-learning in a specific reference of foundational numeracy, through systematic engagement with existing scholarly work.

The literature corpus comprises policy documents, surveys, and reports describing foundational numeracy and its present status in the country; peer-reviewed journal articles, scholarly books and book chapters, authoritative secondary interpretations of Piagetian Theory of cognitive development and its significance in foundational numeracy, stage-appropriate FLN framework and pedagogies in math instruction at early stages.

With a conceptual analysis approach, the analytical process followed the four recursive stages:

- Concept identification: Key Piagetian constructs (e.g. developmental stages, symbolic functionalism, egocentrism, intuitive thought, conservation, classification, irreversibility etc.) and foundational numeracy were identified across selected text.
- Concept comparison: Interpretation and applications of these constructs were compared across the studies.
- Conceptual synthesis: Patterns, convergence, and tensions were synthesised to generate integrative themes.
- Theoretical re-articulation: The synthesised insights were re-articulated to advance a refined conceptual understanding aligned with the objectives of this paper.

To enhance theoretical rigor only peer-reviewed, scholarly work, and authoritative sources were used; and interpretations were grounded in original text wherever possible

Conceptual coherence and logical consistency were maintained throughout the analysis.

Foundational Numeracy

Foundational numeracy is the set of skills and knowledge that children need to develop in order to be successful in

mathematics. It includes understanding basic number concepts, such as counting, addition, subtraction, and place value; being able to solve simple problems; and being able to apply mathematical concepts to real-world situations.

As per NEP (2020), foundational numeracy will comprise of “Numbers, Counting, Shapes, Puzzles, Logical Thinking, Problem-Solving etc.”

Significance of having strong foundational numeracy skills is as follows:



Fig. 1: Benefits of having Strong Numeracy Skills

Developing foundational numeracy is essential for a child’s proper cognitive development but teaching and learning of mathematics at such an early stage is also not so convenient. Now the question arises, how we may improvise education of foundational numeracy so that the learning objectives can be achieved?

process of cognitive development, the theory given by ‘Jean Piaget’ is most widely acceptable. The “Foundational Stage” of learning in NEP (2020) is from 3-8 years of age and this age group majorly falls under the second stage i.e., Pre-Operational Stage of Piaget’s classification of cognitive developmental stages, except for the last year i.e., 8-year-olds, which comes under the third stage, i.e., concrete operational stage.

Foundational Numeracy and Cognitive Development

Among all the theories describing the

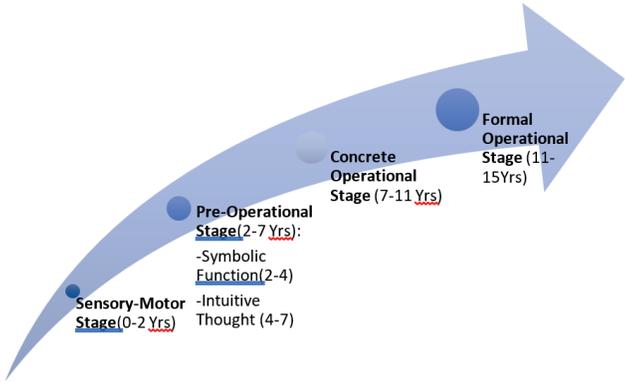


Fig. 2: Piaget’s Cognitive Development Stages

The concept of foundational Numeracy is directly associated with the cognitive domain of learning and it might be better understandable in view of Piaget's

theory of cognitive development. It can be studied into the following two successive stages/levels:

Table - 1: Age groups of foundational stage learning and Piaget's cognitive development stages

Age Group of Foundational Stage learning		Piaget's cognitive development stage	
1.	a. 3 to 4 years	Symbolic Function	Pre- Operational Stage
	b. 4 to 7 years	Intuitive thought	
2.	7 to 8 years	Concrete Operational Stage	

The concept and nature of "Fundamental Numeracy (FN)" at above phases of fundamental stage of learning can be understood in a better way with the characteristics and cognitive abilities acquired at a particular cognitive stage and may be discussed as follows:

1. Symbolic Functionalism and Foundational Numeracy: Piaget

has defined 2 to 4 years of age as 'symbolic function substage'. The term "Symbolic Function" refers to the ability to associate a symbol/word/thing to a particular object/thing. We may represent and understand this exceptional ability of children as a process as follows:



Fig. 3: Symbolic Functioning Process in Children

First, children observe and picture the object in their mind, then they create its mental image/schema to remember it, then they understand it from various perspective (such as its color, shape etc.), then they replicate the idea/mental-image of the object in other times/situations as per need even if the object is not present in front of them. We may say that; it is the age when the ability of "formation of schema" starts developing.

Mathematics is a discipline, where we represent contents, relation, association through various symbols. Numbers themselves are nothing but a symbolic representation of a quantity.

Therefore, we may take advantage of this ability of symbolic-function as a base for learning of fundamental numeracy. Mathematical content (Numbers/Symbols/Shapes etc.) can be presented to children in various interactive and effective ways, so that children will be able to easily associate and assimilate with their cognitive schema and knowledge. According to NCF-FS, "Similar to language, content in mathematics can reflect engagement with the local environment. Mathematical activities, whether understanding shapes or counting, can be integrated with engagement with the natural and human environments."

2. Pretend Play and Foundational Numeracy:

During pre-operational stage children imitate characters (people/animals/cartoons etc.) and try to feel the things in a different way, sometimes, their imagining ability gives life to static objects also, like stick, broom etc. according to their role play. These kind of activities may generate the ability of abstract thinking which is essential for mathematics learning. Also, with role play activities, various numeracy concepts might be introduced well merged with some context or story which is familiar and interesting for kids and capture their attention and involvement. This makes learning more effortless and natural.

3. Egocentrism and Foundational Numeracy:

According to Piaget, 'Egocentrism' refers to children's inability to understand things and context from other's perspective, they only understand and accept things as they see or feel them. Pre-school students may not be able to recognise mathematical concepts in the same way as we do. Therefore, the content must be presented in a way that students find familiar and relevant; and can reflect on them as per their understanding. Teachers must not impose their perspective on students' but let them digest concepts according to their own viewpoint. Teachers only need to observe, guide and correct them as and when required.

For example, to make them recognise different geometrical shapes, the teacher needs to present some concrete objects of different shapes from their surroundings and tell them about their shapes (play-way-method). Teachers will have to repeat this

process and correct them if they make a mistake, until they start recognising particular shapes correctly in other objects also from their surroundings by themselves, i.e., they start reflecting on that. Teachers need to guide their perspective in the right direction instead of imposing their own perspective on them.

4. Perceptual-salience and Foundational Numeracy:

After sensory-motor stage, children start using logic in order to understand the world around them but their logical cognitive ability is not that mature and they still largely depend on their sensory engagement to the objects/things/people around them to understand. 'Recent-sensory-experience' of something hugely affects their perspective, so that, sometimes it overcomes their previous knowledge about that very context. For example, if a person puts on a crown and pretends to be a king while playing with a child, at that moment that person will only be a king to him/her and not a person that he/she previously knew. It is a major limitation of a child's understanding capability in the pre-operational stage and a teacher must keep this in mind while teaching different mathematical concepts. For example, a teacher might find it difficult to make a child understand the similarity of the quantity of three chocolates and the number 3.

5. Animism and Foundational Numeracy:

The term "animism" refers to children's animatic perspective towards different lifeless objects. For example, for them their toys are alive, if they get hurt with a table then table is mean; they talk to their favorite

teddy bear like their best-friend, they ride a broom like a horse, etc. This is where a child starts thinking and imagining things or concepts without any physical reference and this leads to developing the ability of abstract thinking in them. Teachers need to understand this tendency and must design their pedagogy or teaching methods accordingly. They may teach numbers, counting, simple mathematical operations through story-telling or play-way method in which they may construct some mental images of the mathematical concepts being taught. It is an effective way of introducing numeracy in their cognitive process without just rote memorisation. Numbers might also be a child's imaginary friend having their own characteristics, only if the teacher becomes successful in presenting them in that way through play-way method and not through rote memorisation.

6. Intuitive Thought Process and Foundational Numeracy: Piaget called the age of "4-6 years" as "Intuitive thought substage". In this age-group, their thinking becomes less dependent on perception or evidence-based processing thinking and more on their intuitive thinking. This is the time when 'logic' starts taking place in their thinking process. With the development of intuitive thought ability, they try to understand the world around them by observing it minutely with their basic and immature logical reasoning, they also ask a lot of questions about things and incidents in their environment to understand their surroundings better. Asking questions is essential for proper development of cognitive ability, therefore, at this stage the education should

be based on problem-solving and question-answer methods. No matter how illogical or irrelevant children's questions are, educators must show adequate patience and give them logical answers with satisfying explanations and try to satisfy their inquisitive thirst. This way their intuitive thought ability will not only contribute to their cognitive development but they also learn life skills such as problem solving and abstract thinking which are essential skills in mathematics learning at all levels.

7. Centration and Foundational Numeracy: 'Centration' is one of the major limitations of children's intuitive thought ability. 'Centration' refers to children's inability to understand things/objects/phenomena from multiple perspectives simultaneously due to lack of conservation. Their understanding might be significantly affected by some prominent feature(s) of a thing/object/phenomenon i.e., that particular feature/characteristic will become their centre of understanding for that object and they might miss or ignore other 'not so eye catchy' but important details about it. Consequently, it's not possible for them to understand it with multiple important factors at once, since they cannot absorb that much information at once, but if things are being presented one by one and their role and importance are being explained properly, they will be able to store and process that information successfully as per their cognitive capability. In teaching numeracy as well, sometimes, it is possible that children cannot understand the mathematical concept as the teacher wants them to understand since their perspective regarding

that concept might be completely different from the teacher and concentration also limits their understanding. In this condition, if the teacher is not attentive to understand this perceptual difference, the learning will be void and unsuccessful. Therefore, the teacher needs to pay special attention to the student's learning perception, preparing teaching-material and pedagogy while teaching. If the students are missing some important concept, the teacher needs to make them aware of that and direct their perception accordingly for obtaining the desired learning goals positively.

8. Inability of Conservation and Foundational Numeracy: Principle of conservation states that, 'change in shape or rearrangement does not affect the quantity of the matter'. Here Piaget stated that, at the pre-operational stage, the ability of conservation is not present in children, i.e., at this stage they cannot focus/process two or more characteristics/information simultaneously at once. For example, if a child is given same amount of liquid in two jars of different shapes, one is shorter but wider and the other is taller but narrower, and then he asked which jar contain more amount of liquid; then child may choose the taller jar, since he cannot focus on its three dimensions (width, height and volume) simultaneously, he just focuses on one characteristic i.e., height of liquid in the jars and chose the taller jar. This inability of conservation largely affects the learning of numeracy at this stage. Children are unable to handle/understand multiple mathematical operations or

characteristics simultaneously at once. Therefore, it is essential to keep things simple and focused on only one mathematical concept or characteristic at one time and gradually go on higher mathematical concepts through such simpler content specific concepts. Also, appropriate pedagogy must be employed to make learning more effective and fruitful.

9. Classification errors and Foundational Numeracy: Centration and inability of conservation leads classification errors in children at pre-operational stage. Children may find it difficult to understand that things can be classified in multiple ways and classes have subclasses as well. For example, on watching a fruit basket containing bananas, apples, oranges and grapes; the child might say that there are more fruits than bananas, since that child is unable to understand the bananas as a subclass of the larger class of fruits. So, at this stage, in learning mathematics, children may find it difficult to understand different geometrical shapes and their similarity and differences from one another. For example, children may not recognise the difference between circle and oval, square and rectangle or may not classify different geometrical shapes correctly. Also, they follow "Transductive Reasoning", i.e., they infer from a single example and generalise the results. For example, if a child's grandmother wears specs, then for that child all the grandmother must wear specs; or on asking about the occasion of school holiday, whatever it may be but children might name it as a Sunday. In mathematics-learning

the teacher must be careful about classification-errors and must prevent fault inferences. For example, if a student is learning that any number divided by itself, results as 1; in this case the student may refer to it as a general rule and may say that "0 divided by 0 is equal to 1", which is not true. Therefore, the teacher must be careful and guide the students accordingly.

10. Irreversibility and Foundational Numeracy:

At pre-operational stage, a child's perspective and understanding largely depends upon visual representations of objects/ideas. They understand things as they see them at that very moment. Along with centration tendency and lack of conservation ability, irreversibility is also one of the major limitations at this stage. Irreversibility refers to a child's inability to understand a sequence of a process as a whole and return to the initial point in the end. For example, if a child is playing with a dough ball and someone flattens it, that child will not understand that the dough can be reshaped as a ball again. However, according to Piaget, a child passes through this limitation by the age of 7. This limitation of irreversibility can become a big challenge in teaching-learning of numeracy at this stage. The teacher must be well aware and should employ appropriate ways to overcome this limitation in teaching-learning. For example, if a teacher takes jars and fill them with water to teach the concept of quantity, and takes out or add some water to show less or more quantity of water and repeat this process to illustrate the difference between 'more' and 'less', then some students might feel difficult

to recall the initial quantity of water filled in the jar and compare it with the new quantity, and consequently will be unable to understand the concept of more and less quantity. Therefore, it is essential for teachers to monitor students closely while teaching them and guide them accordingly if they are facing such difficulties.

Teaching-Learning of Foundational Numeracy

Mathematics-learning demands constant focus, attention and practice, also sometimes it is abstract in nature and difficult to understand and it is quite difficult to retain such young minds so focused and attentive. Here, we may discuss some effective techniques and pedagogies which might be significantly helpful in improvising teaching-learning of foundational-numeracy at pre-primary and elementary level.

1. Concrete to abstract:

- **Manipulatives:** Building a bridge between the abstract world of numbers and a child's tangible experiences is the key, here, blocks, counters, and other physical objects (manipulatives) become powerful tools. Children may use them to represent numbers visually, understand addition, subtraction through physical movement, and grasp concepts like place-value by building towers with units, tens, and hundreds of blocks.
- **Real-world connections:** Math is not isolated! Make learning relevant by incorporating everyday situations. Counting toys, setting the table, or dividing cookies among friends provides a context for understanding numbers and operations. Grocery shopping becomes a lesson in

addition and subtraction, while comparing heights involves measurement concepts.

2. Playful learning:

- **Math games:** Board games involving counting, dice rolling etc. make practicing math fun and engaging. Card games help focusing on number recognition, sorting, and comparing. Educational apps can also provide a playful learning environment.
- **Movement and songs:** Kinesthetic learners benefit from incorporating physical activity. Clapping along to counting songs, hopping on number lines, or creating number shapes with their bodies turn learning into an active adventure.

3. Building strong number sense:

- **Number talks:** Encourage discussion and exploration, pose thought-provoking questions like “How many apples are on the tree?” or “What number comes after 7?” Let children explain their thinking and listen to different approaches. This builds confidence, fosters problem-solving skills, and strengthens their understanding of number relationships.
- **Subitizing:** Recognising small quantities instantly (subitizing) is a crucial foundation for addition and subtraction. Flashcards with small dot patterns or quick games involving dice rolls can help children develop this vital skill.

4. Differentiation and assessment:

- **Catering to individual needs:** Not all learners progress at the same pace. Regular assessment, understanding

through observations, informal quizzes, student self-reflection allows teachers to differentiate instruction. Some students may need more support with basic counting, while others might be ready for early addition and subtraction problems.

- **Technology integration:** Technology can be a valuable tool. Educational software can provide targeted practice for specific skills, while digital manipulatives offer another avenue for exploring number concepts.

5. Fostering a positive math environment:

- **Celebrate mistakes:** Mistakes are valuable learning opportunities. Create a safe space where students feel comfortable taking risks and exploring different solutions. Celebrate the “aha!” moments that come after overcoming challenges.
- **Positive math language:** Math should be framed as an exciting subject to discover, not a source of anxiety. Use positive and encouraging language, focusing on progress and effort.

By combining these approaches, educators can create a dynamic and engaging learning environment where foundational numeracy thrives. Remember, a strong foundation paves the way for a lifelong love of learning and success in mathematics.

Conclusion

There exists a harmonious alignment between Piaget’s developmental stages and the acquisition of foundational numeracy. By mapping mathematical concepts to a child’s cognitive readiness, educators can ensure that numeracy skills are built on a solid developmental foundation. This relationship underscores that mathematical fluency is not merely

a product of instruction, but a natural progression of cognitive maturity.

Specific cognitive milestones serve as the essential building blocks for mathematical mastery; and to maximise student potential, mathematics instructions should be intentionally tailored to each cognitive stage through

active, discovery-based strategies.

Foundational numeracy is an essential skill for children to develop. By providing children with opportunities to learn and practice math in a variety of ways, parents and educators can help them develop the skills they need to succeed in school and in life.

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