Research Article

Designing and Validating Technological Pedagogical Content Knowledge Strategies for Teaching Mathematics

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Abstract

The ultimatum of the twenty-first century for preparing teachers in technology-rich mathematics classrooms emphasizes an approach in mathematics teaching that makes use of effective choices in teachers' use of technology when teaching a particular content. Technological Pedagogical Content Knowledge (TPCK) is a theoretical knowledge construct enlightening a way to integrate content-specific knowledge and pedagogical strategies while teaching with technology. The major goal of the research was to understand and describe how Technological Pedagogical Content Knowledge Strategies can be designed and validated for guiding mathematics teaching. Technological Pedagogical Content Knowledge Strategies were designed by adopting the methodology of design-based research. These strategies include Technological Pedagogical Content Knowledge Strategies Framework that covers intervention in nine phases for framing the TPCK-based lesson transcript (TPCK Script). The TPCK Strategies were evaluated using the TPCK strategies framework evaluation Proforma. The central theme of this instructional strategy is that it emphasizes the establishment of knowledge–building community learning environments. Considering and applying new instructional strategies can help instructors to understand the uses of pedagogical content knowledge, as well as to reflect the role of technological content knowledge and technological pedagogical knowledge that can be adapted in mathematics teaching in all educational levels and environments.

Keywords: Technological Pedagogical Content Knowledge Strategies, Instructional Strategies, Technological Pedagogical Content Knowledge, Teaching Mathematics, Mathematics Education

Introduction

Significant shifts in the views on teachers' knowledge required for effective teaching have transcended from PCK (Pedagogical Content Knowledge) to a more technically specialized TPCK (Technological Pedagogical Content Knowledge) in the 21st century. Teachers of the 21st century are confronted with expanding their knowledge for teaching with the multitude of recent and emerging technologies, technologies which they have inadequate experience with, much less for integrating them as learning tools in their classrooms. To address this intricacy in the blending of knowledge and experience, they actively search for experiences to assist in reframing their knowledge for teaching with technologies. This knowledge was described as Technological Pedagogical Content Knowledge (TPCK), a dynamic, theoretical construct for designing, implementing, and evaluating curriculum and instruction with technology.

Mathematization of one's thinking is promising by imparting mathematics education. Mathematics Education nurtures cognitive abilities and attitudes

to make life more meaningful. It is a process of human enlightenment and empowerment aiming at achieving a better and higher quality of life. In this era of science and technology, there is a need for more and more mathematical knowledge to confront the challenges of modern technological society. Mathematics has its language with signs, symbols, terms, operations, etc. It helps in drawing conclusions and interpreting various ideas and themes. Mathematics has its tools lay intuition, logic, reasoning, analysis, individuality, generality, and construction, etc.

The teaching of everyday mathematics has become an indispensable part of general education. With the recent emphasis on using technologies in education, particularly in mathematics instruction, the need has emerged to prepare mathematics teachers with effective classroom technology integration skills. The challenge is to design an appropriate strategy for teachers of mathematics by which they can relearn, rethink and redefine teaching and learning as they confront their current conceptions of teaching.

New technologies are advancing into many aspects of our lives, and this progression is evident in the development of technologies to support the teaching and learning of mathematics. A teacher who teaches mathematics should possess the knowledge for integrating different pedagogies and technologies into the content areas. The notion of teachers orchestrating their students' collaboration in mathematics affords new ways to conceptualize teacher pedagogy. Teacher knowledge is never stable, but always changing based on the technologies of the discipline. Therefore, designing strategies grounded in the TPCK framework which promotes teaching with technology as a process of developing knowledge that becomes teacher actions in practice supports the development of content-centric pedagogies for teaching with technology.

Background

Determining an appropriate design for a teaching-learning progression that blends both theoretical and practical experiences in TPCK must draw from multiple research results. Technological Pedagogical Content Knowledge (TPCK) is a supportive knowledge framework for the design of Technological Pedagogical Content Knowledge (TPCK) Strategies. TPCK promotes teachers' teaching content as a complex interaction among content knowledge, pedagogical knowledge and technological knowledge, guiding them in the strategic thinking of when, where, and how to guide students' learning with technologies. TPCK frames teachers' knowledge as the intersection of these three knowledge bases, promoting the intersection as the desirable knowledge upon which teachers rely when designing and implementing curriculum and instruction while guiding students' thinking and learning with technologies in specific content areas.

Technological Pedagogical Content Knowledge (TPCK)

Mishra and Koehler (2006) describe the TPCK as an amalgam of three knowledge bases: content knowledge (CK), pedagogical knowledge (PK) and technological knowledge (TK), in essence indicating the importance of content-specific instructional design. Their model is built upon Shulman's (1986) well-established construct of pedagogical content knowledge (PCK). Koehler and Mishra (2008) conceptualized the TPCK framework as an integration of seven domains of knowledge, which is unique from the other existing individual knowledge forms. They are-

1) Content Knowledge,

2) Pedagogical Knowledge,

3) Technological Knowledge,

4) Pedagogical Content Knowledge,

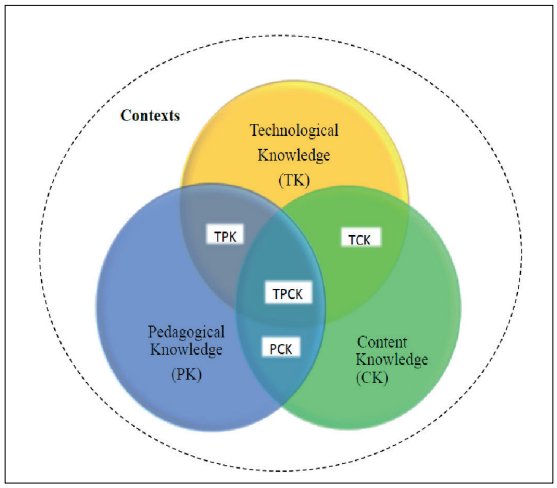
5) Technological Content Knowledge,

6) Technological Pedagogical Knowledge

7) Technological Pedagogical Content Knowledge

Technological Pedagogical Content Knowledge Framework as depicted by Koehler and Mishra (2008) is shown in figure1.

Figure-1: Technological Pedagogical Content Knowledge Framework as depicted by Koehler & Mishra, 2008



The Knowledge Components of TPCK

The seven knowledge components of TPCK are described by Koehler and Mishra (2008) as follows.

1) **Content Knowledge (CK):** Content Knowledge represents the knowledge of the disciplines proposed to be taught or learned. The level of content knowledge varies according to different instructional situations like the differences between the content of mathematics at the school level and graduation level. Content knowledge signifies knowledge of the content area they teach. For example, Knowledge about the content of 8th grade Mathematics.

2) **Pedagogical Knowledge (PK):** Pedagogical Knowledge deals with the knowledge of the methods, strategies, activities, processes, and practices that encourage learning**.** For example, Knowledge about how to use brainstorming in teaching.

3) **Technological Knowledge (TK):** Technological Knowledge refers to the knowledge of affordances of technologies that help in learning. According to Koehler and Mishra (2008), educational technology is all of the tools, techniques, and collective knowledge applicable to education including analog technologies like chalkboard, pencil, and microscope, etc., and digital technologies like the computer, blogging, and internet. For example, Knowledge about how to use the web 2.O tools like wikis, blogs, podcasts, etc.

4) **Pedagogical Content Knowledge (PCK)**: It is the intersection of Pedagogical Knowledge (PK) and Content Knowledge (CK). Some contents cannot be taught to the students using merely an individual pedagogy alone. It emphasizes the importance of the particular type of pedagogies for teaching particular content areas. Pedagogical content knowledge is a term coined by Shulman (1986), which refers to the transformation of content into pedagogically sound forms and to the elucidation of subject matter to be grasped by students in new ways. For example, Knowledge of using discussion methods when teaching a social issue.

5) **Technological Content Knowledge (TCK**): Technological Content Knowledge refers to the combination of Technological Knowledge (TK) with Content Knowledge (CK). Technological Content Knowledge is the technological depiction of content knowledge without any account to teaching (Cox & Graham, 2009). TCK conceptualizes teachers' understanding of how technology can facilitate student skill development in a given subject. That is, teachers must be intimately familiar with the content as well as have the capacity to effectively choose and appropriately leverage technology to support their students' learning. Technology provides extensive representational opportunities for teachers to display content to their students. These representations exist independent of the teacher's knowledge about their use in a pedagogical context. For example, Knowledge about using online dictionaries.

6) **Technological Pedagogical Knowledge (TPK):** It is the interaction of Technological Knowledge (TK) and Pedagogical Knowledge (PK).TPK is the knowledge of using technologies for pedagogical purposes with no reference towards a specific content area. TPK conceptualizes knowledge about how technologies may be used to meet pedagogical aims in the classroom (Koehler & Mishra, 2008).In other words, teachers deeply consider how technologies influence or are influenced, by their pedagogical style and their students' learning styles. TPK is considered to be independent of specific content or topic. It means, TPK can be applied in any content domain. Further, Niess (2008) asserted that teachers with TPK should consider students' learning styles when choosing a particular technology. For example, Knowledge about how to use wikis for collaborative learning.

7) **Technological Pedagogical Content Knowledge (TPCK or TPACK)**: Technological Pedagogical Content Knowledge is the interaction of PCK and TCK and TPK.TPCK is the knowledge required by teachers for using pedagogical techniques that constructively incorporate technologies to teach content. TPCK illustrates teachers' ability to engage in a transactional negotiation among their content, pedagogy, and technology knowledge domains (Mishra & Koehler, 2006). Teachers implement new skills and understandings when they combine these knowledge domains while teaching with technology. Thus, the TPCK construct includes using pedagogical techniques that constructively and continuously incorporate technologies to teach content. For example, Knowledge about how to use YouTube facilitates collaborative learning in teaching the application of physics laws.

Context

Knowledge of the surrounding educational context includes knowledge about the school, the school social networks, students, parental concerns, the available infrastructure, etc. For example, Teachers may be limited to integrating technology into their teaching due to a shortage of technological accessibility.

The researchers identified and proposed Technological Pedagogical Content Knowledge (TPCK) as a path that can be applied in designing Technological Pedagogical Content Knowledge (TPCK) strategies for teaching mathematics with technologies.

Definition of Key Terms

The key terms expressed in the Statement of the problem are defined further.

Technological Pedagogical Content Knowledge (TPCK)

Technological Pedagogical Content Knowledge (TPCK) entails an emergent body of knowledge consisting of three knowledge components of content, pedagogy, and technology and is also supplemented by the interactions among these domains.

Technological Pedagogical Content Knowledge (TPCK) Strategies

Operationally, Technological Pedagogical Content Knowledge Strategies are the instructional strategies involving the integration of three different domains of knowledge of Technological knowledge, Pedagogical Knowledge and Content Knowledge on mutual integration of two it becomes Technological Content Knowledge (TCK), Pedagogical Content Knowledge (PCK), and Technological Pedagogical Knowledge (TPK) and finally, all the three mutually integrates to form Technological Pedagogical Content Knowledge (TPCK).

In the present study, Technological Pedagogical Content Knowledge Strategies include Technological Pedagogical Content Knowledge Strategies Framework and TPCK Script. The researcher benefited from the nine phased Technological Pedagogical Content Knowledge Strategies Framework-based lesson planning for framing lesson transcripts (TPCK Script).

Objectives

1. To design Technological Pedagogical Content Knowledge Strategies (TPCK Strategies) for teaching mathematics
2. To validate the Technological Pedagogical Content Knowledge Strategies (TPCK Strategies) for teaching mathematics

Methodology

The researcher used the design-based research methodology over the three years (2016-2019).In design-based research (DBR), development and research take place through iterative cycles of design, enactment, analysis, and redesign. Edelson (2002) Stated that DBR is conducted "through a parallel and retrospective process of reflection upon the design and its outcomes; the design researchers elaborate upon their initial hypotheses and principles, refining, adding, and discarding-gradually knitting together a coherent theory that reflects their understanding of the design experience".

In DBR, the content, structure, and instructional approaches of intervention are first identified in the analysis and exploration phase of a design project through a literature review and the input of experts and practitioners. This information is then used to design the first iteration of the intervention.

A preliminary literature review is conducted to identify draft design principles that have the potential to address the problem the intervention is being designed to solve.

TPCK Strategies for teaching Mathematics were designed according to three core processes of DBR outlined by McKenny and Reeves (2012) as (a) analysis and exploration, (b) design and construction, and (c) evaluation and reflection and adopted a qualitative approach in DBR. Through design-based research iterative cycles, system prototypes were created with enhanced design features, more sophisticated functionality, and less complexity.

Tools and Materials Designed for the Study

The following tools and materials were designed for the present study.

1. Technological Pedagogical Content Knowledge (TPCK) Strategies Framework (Vrinda Vijayan & Joshith V. P., 2018)
2. TPCK-based lesson transcript (TPCK Script) (Vrinda Vijayan & Joshith V. P.,2018)
3. TPCK Strategies Framework Evaluation Proforma (Vrinda Vijayan & Joshith V. P., 2018)

Description of the Tools and Materials Used

The following tools and materials were very carefully selected for conducting the present study after having a deep study regarding the suitability and appropriateness of the tools. A detailed discussion of every tool used in the present study is given below:

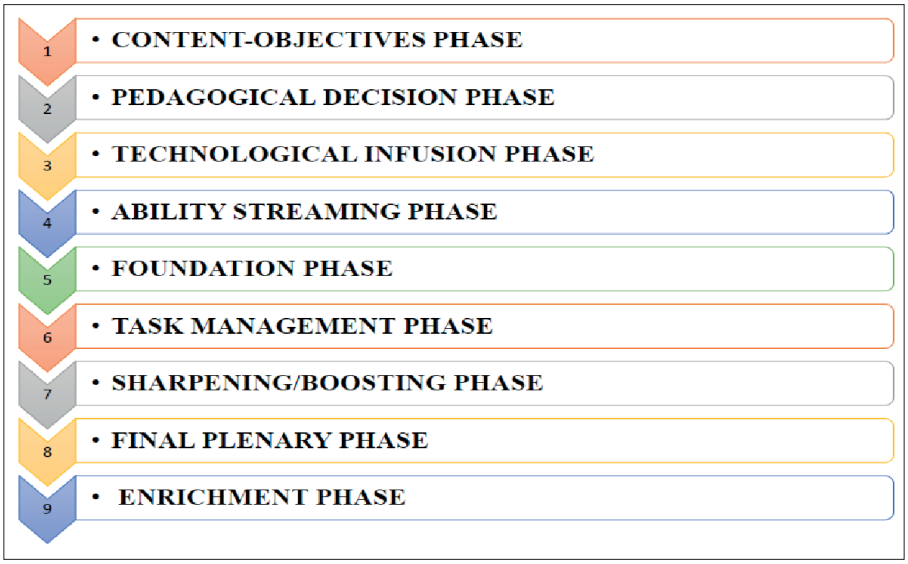
1. **Technological Pedagogical Content Knowledge (TPCK) Strategies Framework**
   Technological Pedagogical Content Knowledge (TPCK) Strategies Framework was designed by the researcher with the guidance of the research supervisor. TPCK Strategies Framework was developed for higher secondary school students of Mathematics. The major aim of preparing the TPCK Strategies Framework was to enhance the mathematical ability of students at higher secondary school in the science stream. It was developed based on the theoretical notions of Punya Mishra and Matthew J. Koehler on Technological Pedagogical Content Knowledge (Koehler & Mishra, 2008). It involves the integration of content, pedagogy, and technology systematically as in five-step curricular-based teaching (Harris & Hofer, 2009).
   The Framework included 33 learning points following certain tasks, which cover a range of topics from Conic Sections, Introduction to Three-dimensional Geometry, Limits, and Derivatives. During the conduct of the experiment, a scheduled amount of forty periods was uniformly fixed to complete the material. It was prepared following ten distinct types of exercises for each learning point: 1) Content 2) Pedagogy 3) Technology 4) Learning Objectives 5) Anticipated mathematical ability 6) Introduction 7) TPCK Narration 8) Pupil Activity 9) Modification 10) Enrichment. The teacher used a variety of pedagogies and technologies merged for teaching the selected content.

Phases of TPCK Strategies Framework

TPCK Strategies Framework was designed to enhance the mathematical ability of higher secondary school students regarding instructional objectives conceptual understanding, procedural knowledge and problem-solving.

TPCK Strategies Framework covers nine phases-Content-Objectives Phase, Pedagogical Decision Phase, Technological Infusion Phase, Ability Streaming Phase, Foundation Phase, Task Management Phase, Sharpening/Boosting Phase, Final Plenary Phase, and Enrichment Phase as shown in figure 2.

Figure-2: Phases of TPCK Strategies Framework



The details of each phase in the development of the Technological Pedagogical Content Knowledge (TPCK) Strategies Framework are given below.

PHASE - I

CONTENT-OBJECTIVES PHASE:
In the first phase of the TPCK Strategies Framework, the teacher selects the content to teach and defines target behavior in terms of the desired behaviour.

i.e.: Content-Objectives Phase includes

(a) Selecting content to teach

(b) Defining the target behavior/ learning outcome of a content

(a) Selecting content to teach: The teacher selects the content area for teaching. For the present study, the teacher selected topics from the chapters' viz. Conic Sections, Introduction to Three-dimensional Geometry, Limits and Derivatives.

(b) Defining the target behavior/ learning outcome of content: The teacher lists out the instructional objectives to be attained at the end of the content area selected.

PHASE - II

Pedagogical Decision Phase: In the second phase of the experiment, the teacher takes decisions regarding the pedagogy to be used and activities to be combined**.**

i.e.: The Pedagogical Decision phase includes

(a) Fix an apt pedagogy

(b) Formulate the activity/ discourse types to combine

(a) Fix an apt pedagogy: The teacher determines the pedagogy that is more suitable to teach the selected topic and to attain the learning objectives. For the present study, the teacher used pedagogies like Demonstration, Simulation, Discussion, Digital Storytelling, Laboratory Method, Collaborative Learning, Inquiry Learning, Seminar and Problem-Solving Tasks, etc

(b) Formulate the activity/discourse types to combine: Teacher plans about suitable activities or discourse types to be included and assures learner participation. The number of tasks and time for each task are set well.

PHASE - III

Technological Infusion Phase: Emerging trends and developments in technology that are most relevant for the topic/content considering its utility and practicability is infused at this phase.

i.e.: Technological Infusion Phase includes

(a) Selecting appropriate technology/software

(b) Evaluating practicability of the technology selected

(a) Selecting appropriate technology/ software: The teacher selects the most appropriate software or technology according to selected pedagogy and content. For each content area, the teacher used different instructional devices like a Laptop and Projector for making use of web 2.O tools, videos, Blooket, Geogebra, and other application software in mathematics.

(b) Evaluating practicability of the technology selected: The teacher verifies whether the selected technology or software is practicable in the actual classroom situation.

PHASE - IV

Ability Streaming Phase: The teacher anticipates mathematical ability to be acquired through the integration of technology, pedagogy, and content.

i.e.: Ability Streaming Phase includes

(a) Mathematical ability to be inculcated

(b) Specifying the anticipated mathematical abilities

(a) Mathematical ability to be inculcated: By teaching a particular content using selected pedagogy and technology, the anticipated mathematical abilities namely conceptual understanding, procedural knowledge and problem-solving formed are identified.

(b) Specifying the anticipated mathematical abilities: The teacher specifies the mathematical abilities for teaching a particular content using different pedagogies and technologies.

PHASE- V

Foundation Phase: At this phase, the teacher directs students about the new way of learning the lesson. The teacher introduces a problematic situation interestingly either visually or orally and assesses the students' entry behaviour.

i.e.: Foundation Phase includes

(a) Orienting the lesson or Induction

(b) Introducing a problematic situation

(c) Testing the entry behaviour

(a) Orienting the lesson or Induction: The teacher orients students about the new way of teaching.

(b) Introducing a problematic situation: The teacher introduces a problematic situation to students in an interesting manner and allows space for students' independent thinking.

(c) Testing the entry behaviour: By the problem presented, the teacher assesses students' previous knowledge in the respective area.

PHASE- VI

Task Management Phase: The phase at which the teacher incorporates the activities by distributing and supplying systematically to those units which have to perform with the decision of the tasks assigned and then presented.

i.e.: Task Management Phase includes

(a) A Presentation focusing on the desired behaviour

(b) Progression/ Distribution of Task

(a) A Presentation focusing on the desired behaviour: The teacher presents the content integrated with suitable pedagogy and technology

(b) Progression/Distribution of Task: The learning progresses with the activities assigned.

PHASE- VII

Boosting/Sharpening Phase:
Boosting is done by adopting the principle of reinforcement. By this, the teacher ensures an enhancement of mathematical ability and then reinforces this with the integration of technological pedagogical content knowledge. Also, the teacher clarifies students' doubts.

i.e.: Boosting Phase includes

(a) Boosting up through TPCK

(b) Reach solution/Sharpening outcome

(a) Boosting up through TPCK: The teacher boosts the students with reinforcement.

(b) Reach solution/Sharpening outcome: Students can solve the problem and ensure the enhancement of mathematical ability.

PHASE- VIII

Final Plenary Phase: In the eighth phase, the newly learned thing becomes an integral part of the learner. This is made clear by feedback, reflection, and evaluation.

i.e.: Final Plenary Phase includes

(a) Assessment /Evaluation

(b) Reflection /Closure/Feedback

(a) Assessment/Evaluation: The teacher measures the attainment of the desired behaviour.

(b) Reflection/Closure/Feedback: Teacher collects feedback from students.

PHASE- IX

Enrichment Phase: In the last phase, the learned behaviour is enriched and followed up by the assignments given.

i.e.: Enrichment Phase includes

(a) Follow-up/Assignments

(b) Resources (more)

(a) Follow-up/Assignments: The learned ability is enriched by assignments or follow-ups given.

(a) Resources: More resources connected with the content are accessed by giving instructions regarding the sources.

Description of phases of TPCK Strategies Framework is given in figure 3.

Figure-3: Description of Phases of TPCK Strategies Framework

2. TPCK Script (TPCK- Based Lesson Transcript)

The researcher prepared lesson transcripts based on TPCK Strategies Framework for the selected chapters from XI standard Mathematics NCERT textbook prescribed for Kerala. It incorporated 33 learning points of 40 lesson scripts. TPCK Script followed the nine phases of the TPCK Strategies framework developed by the investigator. TPCK Scripts were prepared following ten distinct types of exercises for each learning point: 1) Content 2) Learning Objectives 3) Pedagogy 4) Technology 5) Anticipated mathematical ability 6) Introduction 7) TPCK Narration 8) Pupil Activity 9) Modification 10) Enrichment. These exercises are included in the nine phases of the TPCK Strategies Framework.

3. TPCK Strategies Framework Evaluation Proforma

The teacher prepared TPCK Strategies Framework and TPCK script and they were given to the experts for validation. The TPCK Strategies Framework was evaluated using an evaluation proforma. The evaluation proforma for experts was developed by Vijayan, V. & Joshith, V.P., (2018) on a 5-point rating scale. The qualities of good strategies were listed out for preparing the evaluation proforma and the most essential qualities were selected with experts' opinions.

It was developed by considering the following 12 components like Objectives, Content, Pedagogy, Technology, Presentation, Sequencing, Motivation/ Curiosity, Evaluation/ Feedback, Practicability/ Usability, Learner participation, Teacher role, and Integration. The investigator prepared 40 items based on the above aspects. On consultation with experts in the field and the supervising teacher, some items were modified and some were rejected. The final form of evaluation proforma consists of 20 items. The number of items in various components of the evaluation proforma (final) is given in table 1.

Table-1: Break up of Number of items in various components of Evaluation Proforma

Sl. No.	Components	No. of Items
1.	Objectives	1
2.	Content	2
3.	Pedagogy	2
4.	Technology	2
5.	Presentation	3
6.	Sequencing	1
7.	Motivation /Curiosity	1
8.	Evaluation/Feedback	2
9.	Practicability/Usability	2
10.	Learner participation	2
11.	Teacher role	1
12.	Integration	1
	Total	20

Administration of Evaluation Proforma for Validating TPCK Strategies Framework & TPCK Script

The investigator consulted 12 experts including Mathematics teacher educators, research officers, technical experts, higher secondary school teachers, and subject experts to evaluate the developed TPCK Strategies framework and TPCK Script. The evaluation proforma was given to

them for their feedback regarding the framework. The experts were requested to indicate their opinion regarding the framework by putting a tick mark (√) in the appropriate column. The responses were collected and scored. All the Statements were rated on a five-point scale of Excellent / Good / Moderate **/** Satisfactory/ Poor.

Individual Criterion Score

Excellent	= 5 points
Good	= 4 points
Moderate	= 3 points
Satisfactory	= 2 points
Poor	= 1 point

Analysis of TPCK Strategies Evaluation Proforma

The collected data were analysed using percentage analysis. The response patterns of the experts concerning the components are given in table 2.

Table-2: The ratings of experts on the TPCK Strategies Framework (in Percentage)

Sl. No.	Components	Item No.	Expert Rating in Percentage				
			Excellent	Good	Moderate	Satisfactory	Poor
1.	Objectives	1	75	25	0	0	0
2.	Content	2	50	50	0	0	0
		3	41.67	25	33.33	0	0
3.	Pedagogy	4	83.33	8.33	0	8.33	0
		5	33.33	50	16.67	0	0
4.	Technology	6	66.67	33.33	0	0	0
		7	66.67	33.33	0	0	0
5.	Presentation	8	50	41.67	8.33	0	0
		9	58.33	33.33	8.33	0	0
		10	66.67	33.33	0	0	0
6.	Sequencing	11	75	25	0	0	0
7.	Motivation / Curiosity	12	41.67	33.33	25	0	0
8.	Evaluation/ Feedback	13	50	41.67	8.33	0	0
		14	33.33	66.67	0	0	0

Sl. No.	Components	Item No.	Expert Rating in Percentage				
			Excellent	Good	Moderate	Satisfactory	Poor
9.	Practicability/ Usability	15	50	50	0	0	0
		16	58.33	33.33	8.33	0	0
10.	Learner participation	17	33.33	66.67	0	0	0
		18	50	33.33	16.67	0	0
11.	Teacher role	19	33.33	66.67	0	0	0
12.	Integration	20	50	33.33	16.67	0	0

From table 2, it is clear that the observed frequencies of the opinions-Excellent and Good- were very much greater than their counterparts for most of the Statements in the evaluation proforma. So it can be assured that the developed TPCK Strategies satisfy all the criteria of good mathematics learning giving importance to mathematical abilities. The TPCK strategies are flexible and so can be modified according to the requirements of the users if necessary.

Evaluation Report on TPCK Strategies Framework

A report on the evaluation by the experts in the concerned areas on Technological Pedagogical Content Knowledge Strategies was made. The major dimensions considered for evaluation were Objectives, Content, Pedagogy, Technology, Presentation, Sequencing, Motivation/ Curiosity, Evaluation/ Feedback, Practicability/ Usability, Learner participation, Teacher role, and Integration.

The inclusion of learning objectives in the TPCK Strategies Framework and TPCK Script was accomplished. The content selected for teaching using TPCK Strategies Framework with the support of TPCK Script was apt and it was logically progressing from simple to complex or concrete to abstract. Suitable pedagogies for each content area were adopted and facilitated collaborative learning.

Most of the suitable technologies were selected according to the content and animations were processed to provide clarity of concepts. The presentation can provide a visual experience to learners and activities were coined for mathematical abilities; the arrangement of phases was excellent in their opinion. There was also provision for motivation to students. Multiple methods to assess student performance were included in the framework and there is a provision for feedback.

The practicability of the activities included in the framework and usability for mathematics teachers were ensured. It is learner-focused and offers situations for creativity among students' key role for teachers while using the framework. The overall design of the framework was excellent.

The content validity and face validity of the framework and TPCK script were established by the evaluation proforma. Inter-rater reliability (inter judge reliability) refers to the consistency of two or more independent scores, raters, or observers. From Table 1, the consistency in the rating of different experts is verified. Hence the inter-rater

reliability was ensured.

Findings

The iterative nature of the design-based research process over the three years (2016-2019) produced a rich description and clarification of the instructional activities, tasks, forms of interaction, and methods of evaluation supporting the mathematics community of learners as a knowledge-building community in mathematics. Upon the completion of the design for the teaching of mathematics, key instructional strategies emerged in support of the identification of the TPCK strategies. The validity and reliability of the designed TPCK strategies were established.

Features of TPCK Strategies Framework

TPACK Strategies framework covers intervention through nine phases like Content-Objectives Phase, Pedagogical Decision Phase, Technological Infusion Phase, Ability Streaming Phase, Foundation Phase, Task Management Phase, Sharpening/Boosting Phase, Final Plenary Phase and Enrichment Phase. The phases have well defined syntax which can self-direct the teachers to create instructional designs in various domains of learning. The objectives Stated are specific to the context to apply, here the focus is kept on the practical context rather than theoretical notions and the activities listed are properly sequenced with the phases. The teachers' knowledge in managing activities was vital in focusing the strategies in this framework. It involves the integration of content, pedagogy, and technology and their interactions systematically which offers a motivating environment for practice allowing visual demonstrations, interactions to support learning activities. Finally, as an instructional strategy, TPACK is one of the most flexible frameworks in terms of content, pedagogy, and technology.

Discussion and Conclusion

The strategy evolved through research on the design of TPCK in mathematics teaching. The important features of the strategy include intervention on nine phases namely Content-Objectives Phase, Pedagogical Decision Phase, Technological Infusion Phase, Ability Streaming Phase, Foundation Phase, Task Management Phase, Sharpening/ Boosting Phase, Final Plenary Phase, and Enrichment Phase. All phases are clearly defined and systematic with well-defined objectives and properly sequenced activities. The teacher's role is that of a facilitator and classroom management was student-centered. The findings of the research reinforced the importance of instructor actions in creating an environment using technological pedagogical content knowledge where different types of knowledge components are integrated.

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